

PROSPECT THEORY

REFERENCES

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- Tversky, A., and D. Kahneman: Prospect Theory: An Analysis of Decision under Risk, *Econometrica*, Vol. 47, No. 2. (1979), 263-292.
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JUDGEMENT UNDER UNCERTAINTY: HEURISTICS AND BIASES (1974)

- People rely on a limited number of heuristics to reduce complexity in assessing probabilities.
- These heuristics although quite useful might lead to severe and systematic errors.
- Clarity is a heuristic to determine distance. However, if the lights are dim or there is fog, it is likely that you might underestimate the distance away from some object or person.

HEURISTIC 1: REPRESENTATIVENESS

- Probabilities are evaluated by the degree to which A is representative of B.

EXPERIMENT 1

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

(a) Linda is a bank teller.

(b) Linda is a bank teller and is active in the feminist movement.

EXPERIMENT 2

Steve is very shy and withdrawn, invariably helpful, but with little interest in people, or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail.

- (a) Is Steve a farmer?
- (b) Is Steve a salesman?
- (c) Is Steve an airline pilot?
- (d) Is Steve a physician?
- (e) Is Steve a librarian?

EXPERIMENT 3

I will provide a personality description of an individual coming from a sample of 100 professionals. In this sample, there are 70 engineers and 30 lawyers. Indicate the probability that this description belongs to an engineer. Indicate the probability that this description belongs to a lawyer.

Dick is a 30 year old man. He is married with no children. A man of high ability and high motivation, he promises to be quite successful in his field. He is well-liked by his colleagues.

EXPERIMENT 4

A certain town is served by two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know, about 50 percent of all babies are boys. However, the exact percentage varies from day to day. Sometimes it may be higher than 50 percent, sometimes lower. For a period of 1 year, each hospital recorded the days on which more than 60 percent of the babies born were boys. Which hospital do you think recorded more such days?

- (a) The larger hospital?
- (b) The smaller hospital?
- (c) About the same?

EXPERIMENT 5

Which sequence is more likely?

(a) H-T-H-H-T-H

(b) H-H-H-H-H-T

EXPERIMENT 6

You are asked to predict the grades of a second-year student whose first-year record consists of entirely Bs, and one whose first year record consists of an equal number of As and Cs.

Which one do you feel more comfortable predicting?

EXPERIMENT 7

In a discussion of flight training, experienced instructors noted that praise for an exceptionally smooth landing is typically followed by a poorer landing on the next try, while harsh criticism after a rough landing is usually followed by an improvement on the next try. The instructors concluded that verbal rewards are detrimental to learning, while verbal punishments are beneficial, contrary to accepted psychological doctrine.

This conclusion is unwarranted. Why?

REGRESSION TOWARDS THE MEAN IN REPEATED TASKS (GALTON)

Most realistic situations fall between two extremes: for example, one might consider exam scores as a combination of skill and luck. In this case, the subset of students scoring above average would be composed of those who were skilled and had not especially bad luck, together with those who were unskilled, but were extremely lucky. On a re-test of this subset, the unskilled will be unlikely to repeat their lucky break, while the skilled will have a second chance to have bad luck. Hence, those who did well previously are unlikely to do quite as well in the second test.

HEURISTIC 1: REPRESENTATIVENESS

- Insensitivity to prior probability outcomes.
- Insensitivity to sample size.
- Misconception of chance (gambler's fallacy).
- Insensitivity to predictability.
- Illusion of validity.
- Misconception of regression.

HEURISTIC 2: AVAILABILITY

- One may assess the risk of heart attack amongst middle-aged people by recalling such occurrences among one's acquaintances.

EXPERIMENT 8

Subjects heard a list of well-known personalities of both sexes and were subsequently asked to judge whether the list contained more names of men than of women. Different lists were presented to different groups of subjects. In some of the lists the men were relatively more famous than the women, and in others the women were relatively more famous than the men.

In each of the lists, the subjects erroneously judged that the class (sex) that had the more famous personalities was the more numerous.

EXPERIMENT 9

Consider a group of 10 people who form committees of k members, $2 \leq k \leq 8$. How many different committees of k members can be formed?

HEURISTIC 2: AVAILABILITY

- Biases due to the retrievability of instances.
- Biases due to the effectiveness of a search set.
- Biases of imaginability.
- Illusory correlation.

HEURISTIC 3: ADJUSTMENT AND ANCHORING

- People make estimates by starting from an initial value that is adjusted to yield the final answer. The initial value, or starting point, may be suggested by the formulation of the problem, or it may be the result of a partial computation.

EXPERIMENT 10

Subjects were asked to estimate the percentage of African countries in the United Nations. Before indicating their answer, a number between 0 and 100 was determined by spinning a wheel of fortune in the subjects' presence. The subjects were instructed to indicate first whether that number was higher or lower than the percentage, and then to estimate the percentage by moving upwards or downwards from the given percentage.

The median estimates of the percentage of African countries in the United Nations were 25 when the number drawn was 10, and 45 when the number drawn was 65. Payoffs for accuracy did not reduce the anchoring effect.

EXPERIMENT 11

You have a chance to bet on one of three events.

(a) You need to draw a red marble from a bag containing 50% red marbles and 50% white marbles.

(b) You need to draw red marbles 7 consecutive times with replacement from a bag that contains 90% red marbles and 10% white marbles.

(c) You need to draw a red marble at least once in 7 consecutive trials with replacement from a bag containing 10% red marbles and 90% white marbles.

Which event would you choose?

HEURISTIC 3: ADJUSTMENT AND ANCHORING

- Insufficient adjustment
- Biases in the evaluation of conjunctive and disjunctive events.
- Anchoring in the assessment of subjective probability distributions.

PROSPECT THEORY: AN ANALYSIS OF DECISION UNDER RISK (1979)

The Expected Utility framework has been a dominant force in the analysis of decision-making under risk. The framework assumes that all reasonable people would wish to obey its axioms and that most people actually do, most of the time. The present paper describes several classes of choice problems where preferences systematically violate the axioms of Expected Utility framework.

Given these inadequacies, an alternative account of choice under risk is proposed. Prospect Theory assigns value to gains and losses rather than to final assets, and replaces probabilities with decision weights. The value function is normally concave for gains, commonly convex for losses, and is generally steeper for losses than gains.

WHAT ABOUT NEGATIVE PROSPECTS?

Problem 3: $(\$4,000, 0.80) < (\$3,000)$

Problem 4: $(\$4,000, 0.20) > (\$3,000, 0.25)$

Problem 5: $(\$3,000, 0.90) > (\$6,000, 0.45)$

Problem 6: $(\$3,000, 0.002) < (\$6,000, 0.001)$

Problem 3': $(-\$4,000, 0.80) > (-\$3,000)$

Problem 4': $(-\$4,000, 0.20) < (-\$3,000, 0.25)$

Problem 5': $(-\$3,000, 0.90) < (-\$6,000, 0.45)$

Problem 6': $(-\$3,000, 0.002) > (-\$6,000, 0.001)$

THE REFLECTION EFFECT

The preference between negative prospects is the mirror image of the preference between positive prospects. Thus, the reflection of prospects around \$0 reverses the preference order.

This is called the Reflection Effect.

This effect implies that risk aversion in the positive domain is accompanied by risk seeking in the negative domain.

WHAT ABOUT STAGE PROBLEMS?

Problem 10: Consider the following two-stage game. In the first stage, there is a probability of 0.75 to end the game without winning anything, and a probability of 0.25 to move into the second stage. If you reach the second stage you have a choice between

Alternative A: (\$4,000, 0.80)

Alternative B: (\$3,000)

Please indicate your choice.

THE ISOLATION EFFECT

Problem 10 in terms of final outcomes is identical to Problem 4 where subjects preferred $(\$4,000, 0.20)$ over $(\$3,000, 0.25)$. Yet, subjects ignored the first stage and treated the game as Problem 3 where $(\$3,000)$ is preferred over $(\$4,000, 0.80)$.

Subjects often disregard components that the alternatives share, and focus on the components that distinguish them.

This is called the Isolation Effect.

PROSPECT THEORY

Prospect theory distinguishes two phases in the choice process: an early phase of **framing** and a subsequent phase of **evaluation**.

The framing phase consists of a preliminary analysis of the offered prospects, which often yields a simpler representation of these prospects. In the second phase, the edited prospects are evaluated and the prospect of highest values chosen.

π AND v

The overall value of an edited prospect, denoted V , is expressed in terms of two scales, π and v .

The first scale, π , associates with each probability p a decision weight $\pi(p)$, which reflects the impact of p on the overall value of the prospect. However, π is not a probability measure, and it will be shown later that $\pi(p) + \pi(1 - p)$ is typically less than unity. The second scale, v , assigns to each outcome x a number $v(x)$, which reflects the subjective value of that outcome. Recall that outcomes are defined relative to a reference point, which serves as the zero point of the value scale. Hence, v measures the value of deviations from that reference point, i.e., gains and losses.

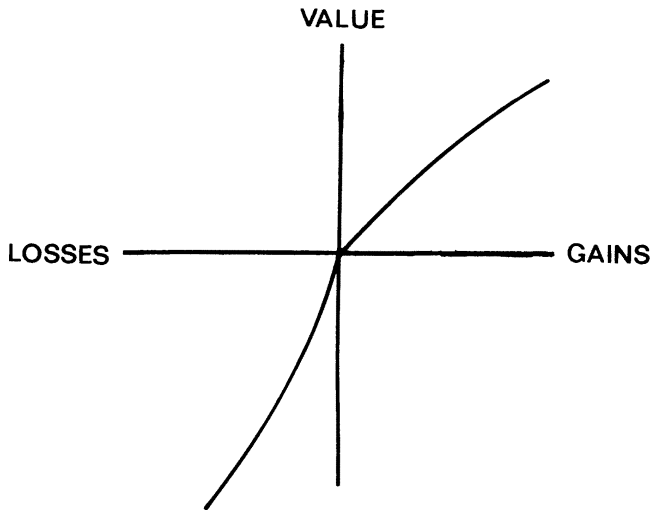
THE VALUE FUNCTION

The value function for changes of wealth is normally concave above the reference point ($v''(x) < 0$ for $x > 0$) and often convex below the reference point ($v''(x) > 0$ for $x < 0$).

In summary, the value function is:

- defined on deviations from the reference point,
- generally concave for gains and commonly convex for losses, and
- steeper for losses than for gains.

A HYPOTHETICAL VALUE FUNCTION

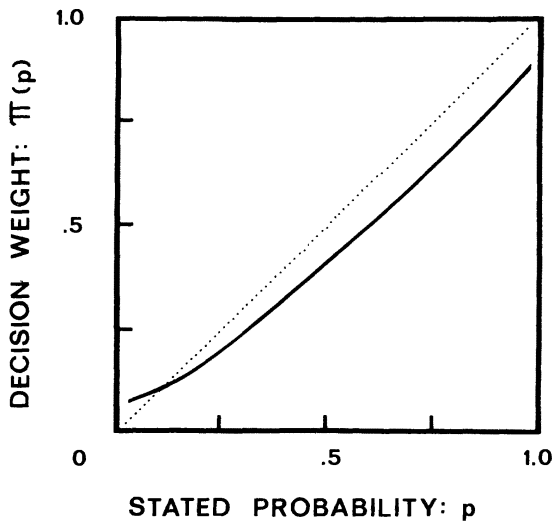


THE WEIGHTING FUNCTION

The value of each outcome is multiplied by a decision weight. Decision weights are not probabilities and should not be interpreted as measures of degree or belief. Decision weights measure the impact of events on the desirability of prospects, and not merely the perceived likelihood of these events.

- $\pi(\cdot)$ is an increasing function of p , with $\pi(0) = 0$ and $\pi(1) = 1$.
- Low probabilities are generally overweighted. That is, $\pi(p) > p$ for small p .
- High probabilities are generally underweighted. That is, $\pi(p) < p$ for high p .

A HYPOTHETICAL WEIGHTING FUNCTION



BASIC EQUATION

If $(x, p; y, q)$ is a regular prospect, then

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y)$$

where $v(0) = 0$, $\pi(0) = 0$, and $\pi(1) = 1$. As in utility theory, V is defined on prospects, while v is defined on outcomes.

ADVANCES IN PROSPECT THEORY: CUMULATIVE REPRESENTATION OF UNCERTAINTY (1992)

A new version of prospect theory is presented that incorporates the cumulative functional and extends the theory to uncertain as well to risky prospects with any number of outcomes. The resulting model, called Cumulative Prospect Theory, combines some of the attractive features of both developments.

WHY CUMULATIVE PROSPECT THEORY?

The weighting scheme used in the original version of prospect theory is a monotonic transformation of outcome probabilities.

- ❶ Such scheme does not always satisfy stochastic dominance.
- ❷ It is hard to extend it to prospects with a large number of outcomes.

Both problems can be solved by the rank-dependent or cumulative functional where instead of transforming each probability separately, the new model transforms the entire cumulative probability distribution function and applies it separately to gains and losses.

FUNCTIONAL FORMS

On one hand, the value functions for gains or losses are given by

$$\tilde{v}(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \\ -\lambda(-x)^\beta & \text{if } x < 0, \end{cases}$$

and on the other hand, the weighting functions for gains or losses are given by

$$w(p) = \begin{cases} \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}} & \text{if } x \geq 0 \\ \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{\frac{1}{\delta}}} & \text{if } x < 0. \end{cases}$$

EXAMPLE

Assume the following parameters:

$$\alpha = 0.88, \beta = 0.88, \lambda = 2.25, \gamma = 0.61, \delta = 0.69.$$

Calculate the cumulative prospect theory value for the prospect where the first outcome is \$300 and the second outcome is -\$100; the probability is 0.3 for the first outcome and 0.7 for the second outcome. Assume the reference point is 0.

The answer is -27.93.